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CP PUBLICATION

SHORT TRICKS

Mathematics

for JEE (Main & Adv)

Sunil Gupta
Career Point Editorial

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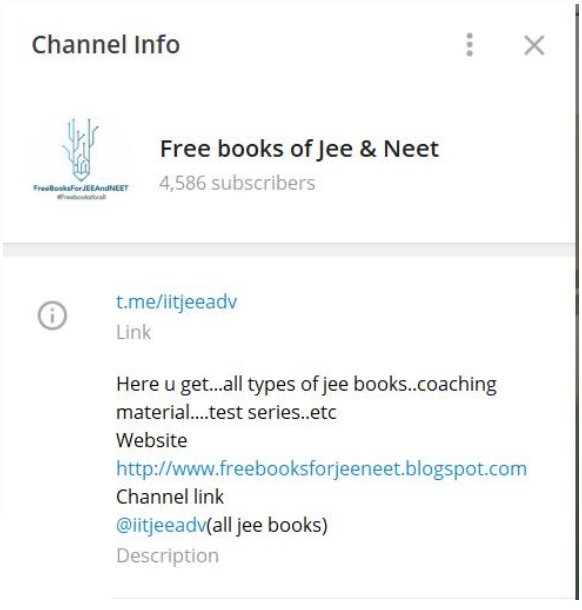
SHORT TRICKS
Mathematics
for JEE (Main & Advanced)



CONTENTS

	Chapter	Page No.
1.	QUADRATIC EQUATION	01
2.	PROGRESSION	15
3.	COMPLEX NUMBER	35
4.	BINOMIAL THEOREM	51
5.	PERMUTATION & COMBINATION	68
6.	DETERMINANT	83
7.	TRIGONOMETRIC RATIO	110
8.	PROPERTIES OF TRIANGLE	134
9.	INVERSE TRIGONOMETRIC FUNCTION	146
10.	POINT & STRAIGHT LINE	159
11.	CIRCLE	184
12.	CONIC SECTIONS	197
13.	VECTOR	227
14.	FUNCTION	244
15.	LIMIT	280
16.	DIFFERENTIATION	288
17.	APPLICATION OF DERIVATIVE	295

9

Inverse Trigonometric Function

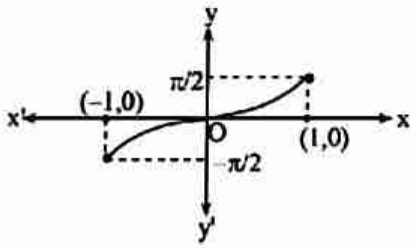
KEY CONCEPTS

1. Inverse Trigonometric Function

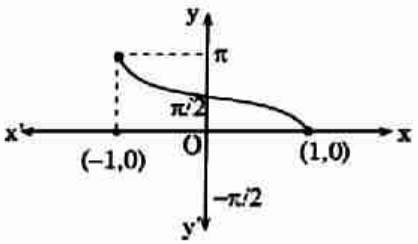
As we know that trigonometric functions are not one-one and onto in their natural domain and range, so their inverse do not exist but if we restrict their domain and range, then their inverse may exists.

2. Graph of Inverse Trigonometric Function

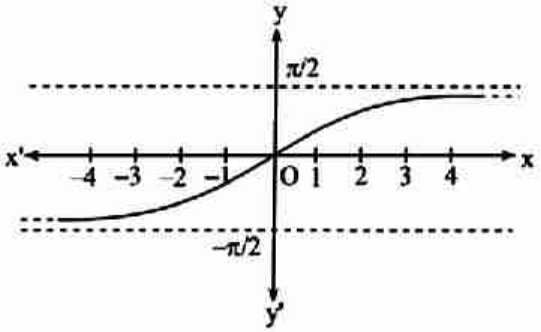
(i) Graph of $y = \sin^{-1}x$



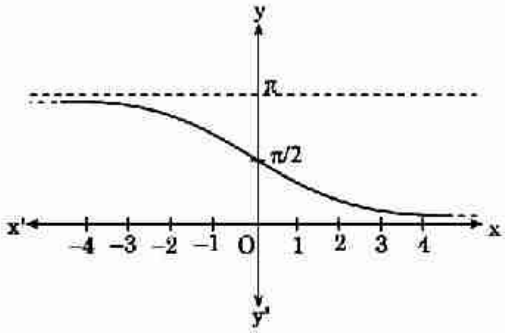
(ii) Graph of $y = \cos^{-1}x$



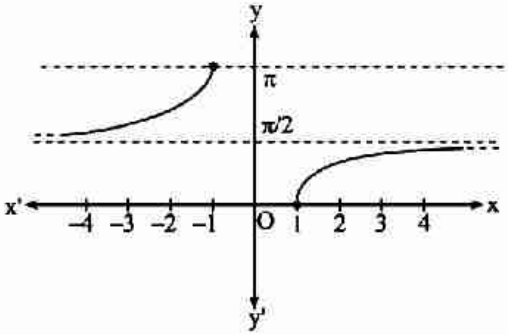
(iii) Graph of $y = \tan^{-1}x$



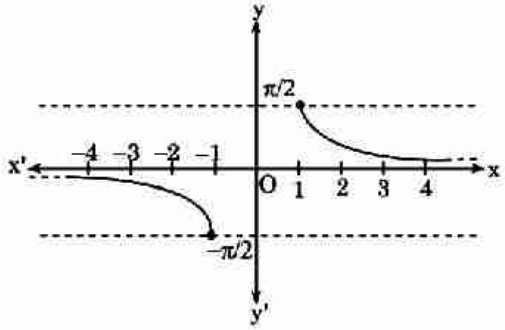
(iv) Graph of $y = \cot^{-1}x$



(v) Graph of $y = \sec^{-1}x$



(vi) Graph of $y = \operatorname{cosec}^{-1}x$



3. Domain and Range of Inverse Trigonometric Functions

Function	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
$\operatorname{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

4. Properties of Inverse Trigonometric Function

Property-1 :

(i) $\sin^{-1}(\sin\theta) = \theta$, provided that $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(ii) $\cos^{-1}(\cos\theta) = \theta$, provided that $0 \le \theta \le \pi$

(iii) $\tan^{-1}(\tan\theta) = \theta$, provided that $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(iv) $\cot^{-1}(\cot\theta) = \theta$, provided that $0 < \theta < \pi$

(v) $\sec^{-1}(\sec\theta) = \theta$, provided that $0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$

(vi) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$, provided that $-\frac{\pi}{2} \le \theta < 0$ or $0 < \theta \le \frac{\pi}{2}$

Property-2 :

(i) $\sin(\sin^{-1}x) = x$, provided that $-1 \le x \le 1$

(ii) $\cos(\cos^{-1}x) = x$, provided that $-1 \le x \le 1$

(iii) $\tan(\tan^{-1}x) = x$, provided that $-\infty < x < \infty$

(iv) $\cot(\cot^{-1}x) = x$, provided that $-\infty < x < \infty$

(v) $\sec(\sec^{-1}x) = x$, provided that $-\infty < x \le -1$ or $1 \le x < \infty$

(vi) $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$, provided that $-\infty < x \le -1$ or $1 \le x < \infty$

Property-3 :

(i) $\sin^{-1}(-x) = -\sin^{-1}x \ \forall\, x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x \ \forall\, x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x \ \forall\, x \in R$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x \ \forall\, x \in R$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x \ \forall\, x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x \ \forall\, x \in (-\infty, -1] \cup [1, \infty)$

Property-4 :

(i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \quad \forall\, x \in [-1, 1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \quad \forall\, x \in R$

(iii) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \ \forall\, x \in (-\infty, -1] \cup [1, \infty)$

Property-5 :

(i) $\sin^{-1}x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right), x \in [-1, 1] \sim \{0\}$ and $\operatorname{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right), x \in (-\infty, -1] \cup [1, \infty)$

(ii) $\cos^{-1}x = \sec^{-1}\left(\frac{1}{x}\right), x \in [-1, 1] \sim \{0\}$ and $\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right), \ x \in (-\infty, -1] \cup [1, \infty)$

(iii) $\tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right), x \in (0, \infty) = -\pi + \cot^{-1}\left(\frac{1}{x}\right), \quad x \in (-\infty, 0)$ and

$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right), x \in (0, \infty) = \pi + \tan^{-1}\left(\frac{1}{x}\right), \quad x \in (-\infty, 0)$

Property-6 :

(i) $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} \pm y\sqrt{1-x^2}]$, if $x, y \ge 0$ and $x^2 + y^2 \le 1$

(ii) $\sin^{-1}x \pm \sin^{-1}y = \pi - \sin^{-1}[x\sqrt{1-y^2} \pm y\sqrt{1-x^2}]$, if $x, y \ge 0$ and $x^2 + y^2 > 1$

Property-7 :

(i) $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}[xy \mp \sqrt{1-x^2}\sqrt{1-y^2}]$, if $x, y > 0$ and $x^2 + y^2 \le 1$

(ii) $\cos^{-1}x \pm \cos^{-1}y = \pi - \cos^{-1}[xy \mp \sqrt{1-x^2}\sqrt{1-y^2}]$, if $x, y > 0$ and $x^2 + y^2 > 1$

Property-8 :

(i) $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right); & \text{if } xy<1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right); & \text{if } x>0, y>0 \text{ and } xy>1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & \text{if } x<0, y<0 \text{ and } xy>1 \end{cases}$

(ii) $\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right); & \text{if } xy>-1 \\ \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right); & \text{if } x>0, y<0 \text{ and } xy<-1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right) & \text{if } x<0, y>0 \text{ and } xy<-1 \end{cases}$

Property-9 :

(i) $2\sin^{-1}x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } \frac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } -1 \le x \le -\frac{1}{\sqrt{2}} \end{cases}$

(ii) $2\cos^{-1}x = \begin{cases} \cos^{-1}(2x^2-1); & \text{if } 0 \le x \le 1 \\ 2\pi - \cos^{-1}(2x^2-1); & \text{if } -1 \le x \le 0 \end{cases}$

(iii) $2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right); & \text{if } -1<x<1 \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right); & \text{if } x>1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right); & \text{if } x<-1 \end{cases}$

Property-10 :

(i) $3\sin^{-1}x = \begin{cases} \sin^{-1}(3x-4x^3); & \text{if } -\frac{1}{2} \le x \le \frac{1}{2} \\ \pi - \sin^{-1}(3x-4x^3); & \text{if } \frac{1}{2} < x \le 1 \\ -\pi - \sin^{-1}(3x-4x^3); & \text{if } -1 \le x < -\frac{1}{2} \end{cases}$

(ii) $3\cos^{-1}x = \begin{cases} \cos^{-1}(4x^3-3x); & \text{if } \frac{1}{2} \le x \le 1 \\ 2\pi - \cos^{-1}(4x^3-3x); & \text{if } -\frac{1}{2} \le x \le \frac{1}{2} \\ 2\pi + \cos^{-1}(4x^3-3x) & \text{if } -1 \le x \le -\frac{1}{2} \end{cases}$

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(iii) $3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right); & \text{if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right); & \text{if } x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right); & \text{if } x < -\frac{1}{\sqrt{3}} \end{cases}$

Property-11 :

(i) $2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right); & \text{if } -1 \le x \le 1 \\ \pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right); & \text{if } x > 1 \\ -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right); & \text{if } x < -1 \end{cases}$

(ii) $2\tan^{-1}x = \begin{cases} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right); & \text{if } 0 \le x < \infty \\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right); & \text{if } -\infty < x < 0 \end{cases}$

Property-12 :

(i) $\sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

(ii) $\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

(iii) $\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}\left(\sqrt{1+x^2}\right) = \operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$

5. Important Points to be remembered

(i) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$ if $x > 0, y > 0, z > 0$ and $(xy + yz + zx) < 1$

(ii) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then $xy + yz + zx = 1$

(iii) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then $x + y + z = xyz$

(iv) $\tan^{-1}x_1 + \tan^{-1}x_2 + \ldots\ldots\ldots + \tan^{-1}x_n = \tan^{-1}\left[\frac{S_1 - S_3 + S_5 - \ldots\ldots\ldots}{1 - S_2 + S_4 - S_6 + \ldots\ldots}\right]$

Where S_k denotes the sum of the product $x_1, x_2, \ldots\ldots\ldots, x_n$ taken k at a time.

(v) $\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right] = \sin^{-1}\left(\frac{x}{a}\right)$

(vi) $\tan^{-1}\left[\frac{3a^2x - x^3}{a(a^2-3x^2)}\right] = 3\tan^{-1}\left(\frac{x}{a}\right)$

(vii) $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

SHORT TRICKS

Trick -1 Method of substitution

Q.1 If $0 \le x \le \frac{1}{2}$, then the value of $\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)-\sin^{-1}x\right)$ is :

(A) 0 (B) -1 (C) 1 (D) $\frac{\pi}{4}$

Sol. [C]

Proper Method

$$\tan\left\{\sin^{-1}\left(\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)-\sin^{-1}x\right\}$$

$$=\tan\left\{\sin^{-1}\left(x\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}+\frac{1}{\sqrt{2}}\sqrt{1-x^2}\right)-\sin^{-1}x\right\}$$

$$=\tan\left\{\left(\sin^{-1}x+\sin^{-1}\frac{1}{\sqrt{2}}\right)-\sin^{-1}x\right\}$$

$$=\tan\left\{\sin^{-1}\frac{1}{\sqrt{2}}\right\}$$

$$=\tan\left(\frac{\pi}{4}\right)=1$$

Short Trick

$\because 0 \le x \le 1/2$, so let $x = 0$

then $\tan\left(\sin^{-1}\left(0+\frac{1}{\sqrt{2}}\right)-\sin^{-1}0\right)=\tan 45^\circ = 1$

Q.2 If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then $x^2 + y^2 + z^2 + 2xyz =$

(A) 1 (B) -1 (C) 0 (D) 2

Sol. [A]

Proper Method

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \cos^{-1}(-z)$

$$\Rightarrow \cos^{-1}\left[xy-\sqrt{1-x^2}\sqrt{1-y^2}\right]=\cos^{-1}(-z)$$

$$\Rightarrow xy-\sqrt{1-x^2}\sqrt{1-y^2}=-z$$

$$\Rightarrow xy+z=\sqrt{1-x^2}\sqrt{1-y^2}$$

$\Rightarrow$ square on both side

$\Rightarrow (xy + z)^2 = (1 - x^2)(1 - y^2)$

$\Rightarrow x^2y^2 + z^2 + 2xyz = 1 - y^2 - x^2 + x^2y^2$

$\Rightarrow x^2 + y^2 + z^2 + 2xyz = 1$

Short Trick

Let $\cos^{-1}x = \cos^{-1}y = \cos^{-1}z = 60^\circ$

Then $x = y = z = \frac{1}{2}$

$\therefore x^2 + y^2 + z^2 + 2xyz$

$$=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+2.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}=1$$

Q.3 If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then which of the following is true?

(A) $x + y + z + xyz = 0$ (B) $x + y + z = xyz$
(C) $xy + yz + zx + 1 = 0$ (D) $xy + yz + zx = 1$

Sol. [D]

Proper Method

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left\{\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right\} = \frac{\pi}{2}$

$\Rightarrow \frac{x+y+z-xyz}{1-(xy+yz+zx)} = \infty$

$\Rightarrow 1-(xy+yz+zx) = 0$

$\Rightarrow xy + yx + zx = 1$

Short Trick

$\therefore$ let $\tan^{-1}x = \tan^{-1}y = \tan^{-1}z = 30°$

$\Rightarrow x = y = z = \frac{1}{\sqrt{3}}$

after putting the values of x, y, z in the option (D) will give the correct answer

Q.4 If $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \theta$ and if $9x^2 - 12xy\cos\theta + 4y^2 = k^2\sin^2\theta$, then k =

(A) ± 3 (B) ± 1 (C) ± 6 (D) ± 2

Sol. [C]

Proper Method

$\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$

$\Rightarrow \cos^{-1}\left(\frac{x}{2}\cdot\frac{y}{3} - \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}\right) = \theta$

$\Rightarrow \frac{xy}{6} - \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}} = \cos\theta$

$\Rightarrow \left(\frac{xy}{6} - \cos\theta\right)^2 = \left(1-\frac{x^2}{4}\right)\left(1-\frac{y^2}{9}\right)$

$\Rightarrow \frac{x^2y^2}{36} + \cos^2\theta - \frac{xy}{3}\cos\theta = 1 - \frac{y^2}{9} - \frac{x^2}{4} + \frac{x^2y^2}{36}$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} - \frac{xy}{3}\cos\theta = 1 - \cos^2\theta$

$\Rightarrow 9x^2 + 4y^2 - 12xy\cos\theta = 36\sin^2\theta$

$\therefore k^2 = 36 \Rightarrow k = \pm 6$

Short Trick

Let $x = 1$ and $y = 0$, then $\theta = 150°$

$\Rightarrow 9x^2 - 12xy\cos\theta + 4y^2 = k^2\sin^2\theta$

$\Rightarrow 9 \times 1 - 0 + 0 = k^2\left(\frac{1}{2}\right)^2$

$\Rightarrow k^2 = 36 \Rightarrow k = \pm 6$

Q.5 If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ and if $x^4 + y^4 + z^4 + 4x^2y^2z^2 = k(x^2y^2 + y^2z^2 + z^2x^2)$ then the value of k is :

(A) 2 (B) 3 (C) 1 (D) 4

Sol. [A]

Proper Method

$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$

$\Rightarrow \sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}z$

$\Rightarrow \cos^{-1}\sqrt{1-x^2} + \cos^{-1}\sqrt{1-y^2} = \pi - \cos^{-1}\sqrt{1-z^2}$

$\Rightarrow \cos^{-1}\left[\sqrt{1-x^2}\sqrt{1-y^2} - xy\right] = \cos^{-1}(-\sqrt{1-z^2})$

$\Rightarrow \sqrt{1-x^2}\sqrt{1-y^2} - xy = -\sqrt{1-z^2}$

Short Trick

$\because \sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$

$\therefore$ Let $\sin^{-1}x = 0° \Rightarrow x = 0$, $\sin^{-1}y = 90°$

$\Rightarrow y = 1$, $\sin^{-1}z = 90° \Rightarrow z = 1$

Put the values of x, y, z in the given expression then we will get $k = 2$.

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$\Rightarrow \sqrt{1-x^2}\sqrt{1-y^2} = xy - \sqrt{1-z^2}$
Square on both side
$\Rightarrow 1 - y^2 - x^2 + x^2y^2 = x^2y^2 + 1 - z^2 - 2xy\sqrt{1-z^2}$
$\Rightarrow x^2 + y^2 - z^2 = 2xy\sqrt{1-z^2}$
again square
$\Rightarrow x^4 + y^4 + z^4 + 2(x^2y^2 - y^2z^2 - z^2x^2) = 4x^2y^2(1 - z^2)$
$\Rightarrow x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$
$k = 2$

Q.6 Let $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$ then $a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{(1-c^2)} =$

(A) 2abc (B) abc (C) $\frac{1}{2}$abc (D) $\frac{abc}{3}$

Sol. **[A]**

Proper Method
Let $A = \sin^{-1}a$, $B = \sin^{-1}b$, $C = \sin^{-1}c$, then
$\sin A = a$, $\sin B = b$, $\sin C = c$
Also as given $A + B + C = \pi$
Further we know that if $A + B + C = \pi$, then
$\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$ (identity result)
$\Rightarrow 2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C = 4\sin A \sin B \sin C$
$\Rightarrow a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2} = 2abc$

Short Trick
Let $\sin^{-1}a = \sin^{-1}b = \sin^{-1}c = 60°$
$\Rightarrow a = b = c = \frac{\sqrt{3}}{2}$
L.H.S. $= a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2}$
$= \frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}} + \frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}} + \frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}} = \frac{3\sqrt{3}}{4}$
after putting the values of a, b, c in the options (A) will give the R.H.S.

Q.7 If $x^2 + y^2 + z^2 = k^2$, then value of $\tan^{-1}\left(\frac{xy}{zk}\right) + \tan^{-1}\left(\frac{xz}{yk}\right) + \tan^{-1}\left(\frac{zy}{xk}\right)$ is equals to ?

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{3\pi}{2}$ (D) 0

Sol. **[A]**

Proper Method
$x^2 + y^2 + z^2 = k^2$
$= \tan^{-1}\left(\frac{xy}{zk}\right) + \tan^{-1}\left(\frac{xz}{yk}\right) + \tan^{-1}\left(\frac{zy}{xk}\right)$
$= \tan^{-1}\left[\frac{\frac{xy}{zk} + \frac{xz}{yk} + \frac{zy}{xk} - \left(\frac{xy}{zk}\right)\left(\frac{xz}{yk}\right)\left(\frac{zy}{xk}\right)}{1 - \left\{\left(\frac{xy}{zk}\right)\left(\frac{xz}{yk}\right) + \left(\frac{xz}{yk}\right)\left(\frac{zy}{xk}\right) + \left(\frac{zy}{xk}\right)\left(\frac{xy}{zk}\right)\right\}}\right]$
$= \tan^{-1}\left[\frac{\left(\frac{xy}{zk}\right) + \left(\frac{xz}{yk}\right) + \left(\frac{zy}{xk}\right) - \left(\frac{xy}{zk}\right)\left(\frac{xz}{yk}\right)\left(\frac{zy}{xk}\right)}{1 - \left(\frac{x^2+y^2+z^2}{k^2}\right)}\right]$
$= \tan^{-1}(\infty)$
$= \frac{\pi}{2}$

Short Trick
Let $x = y = z = 1$, then $k^2 = 3 \Rightarrow k = \sqrt{3}$
$\therefore \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
$= 30° + 30° + 30° = 90° = \frac{\pi}{2}$

Q.8 The value of $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{x}{y}\right)\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{x}{y}\right)\right\}$ is equals to ?

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$ (C) $\frac{2x}{y}$ (D) $\frac{2y}{x}$

Sol. [D]

Proper Method

$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{x}{y}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{x}{y}\right\}$

Let $\frac{1}{2}\cos^{-1}\frac{x}{y}=\theta$

$\Rightarrow \cos^{-1}\frac{x}{y}=2\theta$

$\Rightarrow \cos 2\theta=\frac{x}{y}$

$=\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)$

$=\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}$

$=\frac{2(1+\tan^2\theta)}{1-\tan^2\theta}$

$=\frac{2}{\cos 2\theta}=\frac{2y}{x}$

Short Trick

Let $x = 1$ and $y = 2$ $\left(\because \cos^{-1}\left(\frac{1}{2}\right)=30^\circ\right)$

$\therefore$ L.H.S $=\tan\left\{\frac{\pi}{4}+\frac{1}{2}.30^\circ\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}.30^\circ\right\}$

$=\tan 75^\circ+\tan 15^\circ=\cot 15^\circ+\tan 15^\circ$

$=2+\sqrt{3}+2-\sqrt{3}=4$

Now put $x = 1$ and $y = 2$ in options then (D) will give 4 and (D) is right choice.

Q.9 If $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$, then $4x^2-4xy\cos\alpha+y^2$ is equal to :

(A) $4\sin^2\alpha$ (B) $-4\sin^2\alpha$ (C) $2\sin 2\alpha$ (D) 4

Sol. [A]

Proper Method

$\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$

$\Rightarrow \cos^{-1}\left\{\frac{xy}{2}-\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}\right\}=\alpha$

$\Rightarrow \frac{xy}{2}-\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}=\cos\alpha$

$\Rightarrow \left(\frac{xy}{2}-\cos\alpha\right)^2=(1-x^2)\left(1-\frac{y^2}{4}\right)$

$\Rightarrow \frac{x^2y^2}{4}+\cos^2\alpha-xy\cos\alpha=1-\frac{y^2}{4}-x^2+\frac{x^2y^2}{4}$

$\Rightarrow x^2+\frac{y^2}{4}-xy\cos\alpha=1-\cos^2\alpha$

$\Rightarrow 4x^2+y^2-4xy\cos\alpha=4\sin^2\alpha$

Short Trick

Let $x = 0$ and $y = 1$

Then $\alpha=\cos^{-1}0-\cos^{-1}\left(\frac{1}{2}\right)$

$=90^\circ-60^\circ=30^\circ$

L.H.S. $=4x^2-4xy\cos\alpha+y^2=0+0+1=1$

Now put the value of α in options then only option (A) will give match with L.H.S.

Q.10 If a, b, c are positive real numbers and

$\theta = \tan^{-1}\left(\sqrt{\frac{a(a+b+c)}{bc}}\right) + \tan^{-1}\left(\sqrt{\frac{b(a+b+c)}{ca}}\right) + \tan^{-1}\left(\sqrt{\frac{c(a+b+c)}{ab}}\right)$, then $\tan\theta =$

(A) 1 (B) 0 (C) $\frac{a+b+c}{abc}$ (D) $\frac{abc}{a+b+c}$

Sol. **[B]**

Proper Method

$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

Let $\frac{a+b+c}{abc} = x^2$

then $\theta = \tan^{-1} ax + \tan^{-1} bx + \tan^{-1} cx$

$\theta = \tan^{-1}\left\{\frac{(ax+bx+cx)-(abcx^3)}{1-(abx^2+bcx^2+cax^2)}\right\}$

$\Rightarrow \tan\theta = \left[\frac{x\left\{(a+b+c)-abc\left(\frac{a+b+c}{abc}\right)\right\}}{1-(abx^2+bcx^2+cax^2)}\right]$

$\Rightarrow \tan\theta = 0$

Short Trick

Put a = b = c = 1, then

$0 = \tan^{-1}(\sqrt{3}) + \tan^{-1}(\sqrt{3}) + \tan^{-1}(\sqrt{3})$

$\Rightarrow \theta = 3\tan^{-1}(\sqrt{3}) = 3.\frac{\pi}{3} = \pi$

Q.11 If $S_n = \tan^{-1}\left(\frac{x}{x^2+1.2}\right) + \tan^{-1}\left(\frac{x}{x^2+2.3}\right) + \tan^{-1}\left(\frac{x}{x^2+3.4}\right)$..... n terms, then $S_n =$

(A) $\tan^{-1}\left(\frac{n+1}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right)$ (B) $\tan^{-1}\left(\frac{n-1}{x}\right) + \tan^{-1}\left(\frac{1}{x}\right)$

(C) $\tan^{-1}\left(\frac{n+1}{x}\right) + \tan^{-1}\left(\frac{1}{x}\right)$ (D) $\tan^{-1}\left(\frac{n-1}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right)$

Sol. **[A]**

Proper Method

$f(x) = \tan^{-1}\left(\frac{\frac{2}{x}-\frac{1}{x}}{1+\frac{1}{x}.\frac{2}{x}}\right) + \tan^{-1}\left(\frac{\frac{3}{x}-\frac{2}{x}}{1+\frac{3}{x}.\frac{2}{x}}\right)$

$+ \tan^{-1}\left(\frac{\frac{4}{x}-\frac{3}{x}}{1+\frac{4}{x}.\frac{3}{x}}\right)$ ..... $\tan^{-1}\left(\frac{\frac{n+1}{x}-\frac{n}{x}}{1+\frac{n+1}{x}.\frac{n}{x}}\right)$

$f(x) = \tan^{-1}\left(\frac{2}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{3}{x}\right)$

$- \tan^{-1}\left(\frac{2}{x}\right)$ ..... $+ \tan^{-1}\left(\frac{n+1}{x}\right) - \tan^{-1}\left(\frac{n}{x}\right)$

$f(x) = \tan^{-1}\left(\frac{n+1}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right)$

Short Trick

Put $n = 1 \Rightarrow S_1 = \tan^{-1}\left(\frac{x}{x^2+2}\right)$

by option (A) is correct answer

Q.12 The value of $\tan^{-1}\left(\frac{x}{1+2x^2}\right)+\tan^{-1}\left(\frac{x}{1+6x^2}\right)+\tan^{-1}\left(\frac{x}{1+12x^2}\right)+\ldots\ldots$ n terms is :

(A) $\tan^{-1}(n+1)x-\tan^{-1}x$ (B) $\tan^{-1}(n+1)x+\tan^{-1}x$

(C) $\tan^{-1}(n-1)x-\tan^{-1}x$ (D) $\tan^{-1}(n-1)x+\tan^{-1}x$

Sol. **[A]**

Proper Method

$\because T_n=\tan^{-1}\left(\frac{x}{1+n(n+1)x^2}\right)$

$=\tan^{-1}\left(\frac{(n+1)x-nx}{1+(n+1)x.nx}\right)=\tan^{-1}((n+1)x)-\tan^{-1}(nx)$

$\therefore T_n=\tan^{-1}((n+1)x)-\tan^{-1}(nx)$

$T_1=\tan^{-1}2x-\tan^{-1}x$

$T_2=\tan^{-1}3x-\tan^{-1}2x$

$T_3=\tan^{-1}4x-\tan^{-1}3x$

.....

.....

$T_n=\tan^{-1}(n+1)x-\tan^{-1}nx$

$\therefore S_n=T_1+T_2+T_3\ldots\ldots+T_n=\tan^{-1}(n+1)x-\tan^{-1}x$

Short Trick

Put $n=1\Rightarrow\tan^{-1}\left(\frac{x}{1+2\times2}\right)$

by option (A) is correct answer

Trick -2 Combination of method of substitution and balancing

Q.13 The value of $\cos^{-1}\left(\frac{2+3\cos x}{3+2\cos x}\right)$ is :

(A) $\tan^{-1}\left(\frac{1}{\sqrt{5}}\tan\left(\frac{x}{2}\right)\right)$ (B) $\tan^{-1}\left(\frac{1}{2\sqrt{5}}\tan\left(\frac{x}{2}\right)\right)$

(C) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\tan\left(\frac{x}{2}\right)\right)$ (D) $\tan^{-1}\left(\frac{\sqrt{5}}{2}\tan x\right)$

Sol. **[C]**

Proper Method

$y=\cos^{-1}\left(\frac{2+3\cos x}{3+2\cos x}\right)$

$y=\cos^{-1}\left[\frac{2+3\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)}{3+2\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)}\right]$

$y=\cos^{-1}\left[\frac{5-\tan^2 x/2}{5+\tan^2 x/2}\right]$

$y=\tan^{-1}\left[\frac{\sqrt{20}\tan x/2}{5-\tan^2 x/2}\right]$

$y=\tan^{-1}\left[\frac{2\left(\frac{1}{\sqrt{5}}\tan x/2\right)}{1-\left(\frac{1}{\sqrt{5}}\tan x/2\right)^2}\right]$

$y=2\tan^{-1}\left[\frac{1}{\sqrt{5}}\tan\frac{x}{2}\right]$

Short Trick

Put $x=90°$, then L.H.S.

$=\cos^{-1}\left(\frac{2+0}{3+0}\right)=\cos^{-1}\left(\frac{2}{3}\right)=\tan^{-1}\left(\frac{\sqrt{5}}{2}\right)$

after putting $x=90°$ in the options, (C) will given required result.

Q.14 The value of $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)$ is :

(A) $\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$ (B) $\cos^{-1}\left(\frac{a\cos\theta}{a\cos\theta+b}\right)$ (C) $\cos^{-1}\left(\frac{a\cos\theta}{a+b\cos\theta}\right)$ (D) $\cos^{-1}\left(\frac{b\cos\theta}{a\cos\theta+b}\right)$

Sol. **[A]**

Proper Method

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)$$

$$= \cos^{-1}\left(\frac{1-\frac{a-b}{a+b}\tan^2\frac{\theta}{2}}{1+\frac{a-b}{a+b}\tan^2\frac{\theta}{2}}\right)$$

$$= \cos^{-1}\left[\frac{(a+b)-(a-b)\tan^2\frac{\theta}{2}}{(a+b)+(a-b)\tan^2\frac{\theta}{2}}\right]$$

$$= \cos^{-1}\left[\frac{b\left(1+\tan^2\frac{\theta}{2}\right)+a\left(1-\tan^2\frac{\theta}{2}\right)}{a\left(1+\tan^2\frac{\theta}{2}\right)+b\left(1-\tan^2\frac{\theta}{2}\right)}\right]$$

divide by $\left(1+\tan^2\frac{\theta}{2}\right)$

$$= \cos^{-1}\left[\frac{b+a\cos\theta}{a+b\cos\theta}\right]$$

Short Trick

Let $a = 2$, $b = 1$ and $\theta = 90°$

then L.H.S. $= 2\tan^{-1}\left(\sqrt{\frac{2-1}{2+1}}\tan\left(\frac{90°}{2}\right)\right)$

$= 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\times 1\right) = 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 2\times 30° = 60°$

put the values of a, b and θ in the options (A) will give the required result.

Q.15 If $a_1, a_2, a_3, \ldots\ldots a_n$ are in A.P. with common difference d, then

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$ is equal to -

(A) $\frac{nd}{1+a_1a_n}$ (B) $\frac{a_n-a_1}{a_n+a_1}$ (C) $\frac{(n-1)d}{a_1+a_n}$ (D) $\frac{(n-1)d}{1+a_1a_n}$

Sol. **[D]**

Proper Method

If exp. = tan(S), then

$$S = \tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)+\ldots.+\tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)$$

$$= (\tan^{-1}a_2-\tan^{-1}a_1)+(\tan^{-1}a_3-\tan^{-1}a_2)+\ldots.+(\tan^{-1}a_n-\tan^{-1}a_{n-1})$$

$$= -\tan^{-1}a_1+\tan^{-1}a_n$$

$$= \tan^{-1}\left(\frac{a_n-a_1}{1+a_1a_n}\right) = \tan^{-1}\frac{(n-1)d}{1+a_1a_n}$$

$$\therefore \text{ Exp.} = \tan\left[\tan^{-1}\frac{(n-1)d}{1+a_1a_n}\right] = \frac{(n-1)d}{1+a_1a_n}$$

Short Trick

Let $n = 2$

$$\tan\left[\tan^{-1}\frac{d}{1+a_1a_2}\right] = \frac{d}{1+a_1a_2} =$$

Hence (D) is correct option

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Q.16 If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi/2$, then $x^2 + y^2 + z^2 + 2xyz$ is equal to

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{2}$

Sol. **[B]**

Proper Method

From given relation

$\sin^{-1}x + \sin^{-1}y = \pi/2 - \sin^{-1}z$

$\Rightarrow \cos^{-1}\sqrt{1-x^2} + \cos^{-1}\sqrt{1-y^2} = \cos^{-1}z$

$\Rightarrow \cos^{-1}\left[\sqrt{1-x^2}\sqrt{1-y^2} - xy\right] = \cos^{-1}z$

$\Rightarrow \sqrt{1-x^2}\sqrt{1-y^2} - xy = z$

$\Rightarrow (1-x^2)(1-y^2) = (xy+z)^2$

$\Rightarrow x^2 + y^2 + z^2 + 2xyz = 1$

Short Trick

Let $\sin^{-1}x = 0$, $\sin^{-1}y = 30°$, $\sin^{-1}z = 60°$

Then value of expression $= \frac{1}{4} + \frac{3}{4} = 1$

Q.17 $\cot^{-1}\left(\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right)$ is equal to

(A) $x/2$ (B) $\pi - x/2$ (C) $\pi - x$ (D) $2\pi - x$

Sol. **[B]**

Proper Method

$\text{Exp.} = \cot^{-1}\left[\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}} \cdot \frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} + \sqrt{1+\sin x}}\right]$

$= \cot^{-1}\left(\frac{1+\cos x}{-\sin x}\right)$

$= \cot^{-1}\left(-\frac{2\cos^2 x/2}{2\sin x/2\cos x/2}\right)$

$= \cot^{-1}(-\cot x/2)$

$= \cot^{-1}[\cot(\pi - x/2)]$

$= \pi - x/2$

Short Trick

Let $x = \frac{\pi}{2}$

$\Rightarrow \cot^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right) = \cot^{-1}(-1)$

$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Hence (B) is correct option

Q.18 If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$ is equal to

(A) xyz (B) $1/xyz$ (C) 1 (D) 0

Sol. **[C]**

Proper Method

$\tan^{-1}x + \tan^{-1}y = \pi - \tan^{-1}z$

$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}(-z)$

$\Rightarrow \frac{x+y}{1-xy} = -z$

$\Rightarrow x + y + z = xyz$

$\Rightarrow \frac{1}{yz} + \frac{1}{zx} + \frac{1}{xy} = 1$

Short Trick

Let $\tan^{-1}x = \tan^{-1}y = \tan^{-1}z = \frac{\pi}{3}$

$\therefore x = y = z = \sqrt{3}$

So, $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$